Superconducting Magnets for M.H.D. Generators [and Discussion]

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XVIII. Superconducting magnets for m.h.d. generators

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This paper will review the science and technology of superconducting magnets and the possibility of their application in large m.h.d. generators. The newly exploited hard superconductors offer, in principle, the most economic means of providing magnetic fields of several tens of kilo-oersteds over large volumes. Unfortunately these materials have exhibited certain peculiarities of behaviour when wound into coils ('training' and 'degradation') which, until recently, had made it impossible to realize their full potential or to design larger coils with any confidence. The reasons for this are now fairly well understood and are seen to be fundamental to hard superconductors. A number of techniques have been developed to reduce these effects, the most recent, involving the intimate combination of the superconductor with a metal of low resistivity together with the provision of adequate cooling, circumvents the problem completely.

A further problem with large superconducting magnets is to protect the windings, in the event of a coolant failure, against destruction by the release of the stored magnetic energy, 10^{10} J in the case of a large m.h.d. generator magnet. It will be shown that this also dictates the use of composite conductors in which the superconductor is the minor component.

Although a number of detailed problems of cryogenic engineering remain to be solved, it is now possible to design windings for large superconducting magnets that will operate predictably, safely and at the full potential of the superconductor.

Introduction

The provision of the magnetic field for a large m.h.d. generator poses major problems. Design studies, at present under way in the C.E.G.B., indicate that a generator with a thermal input of 2 GW will require a field of the order of 60 kG over a working volume some 15 m long by 5 m high by 5 m wide. To allow passage of the combustion products into and out of the generating duct, the magnet windings will take a saddle form such as that shown in figure 1. The magnetic stored energy in such a magnet will be greater than 10^{10} J, equivalent to the output of the entire National Grid flowing for 0·3 s or so.

To provide such a field an electromagnet is mandatory and three types of winding can be considered. The first uses conventional water-cooled copper conductors with high power consumption. In the second, the power consumption is reduced considerably by refrigerating the windings to low temperature to improve their conductivity. This reduction more than offsets the additional power consumption of the refrigerator, particularly in the case of metals which can be prepared in a pure state and which show a low magnetoresistance. Aluminium and sodium appear to be the best choices (Post & Taylor 1960, 1962) and for each there is an optimum temperature determined by the balance between increasing electrical conductivity and decreasing thermodynamic efficiency of refrigeration as the temperature is lowered. For aluminium of the highest practical purity this optimum is close to 20 °K, while for sodium it lies close to 8 °K. Small scale experience exists with both systems (Purcell 1962; Laurence & Brown 1962; Taylor & Nelson 1963).

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The third possibility is to use superconducting windings operating close to 4 °K with essentially no power consumption. Given equal technical feasibility, the choice between these three approaches must be made on economic grounds.

It is only in the last five years that superconducting windings have become a real possibility. Hitherto it had been accepted that the critical fields of known materials would limit their useful application to the range below 10 kG. We now know that superconducting materials, such as the compounds Nb_3Sn and V_3Ga and alloys Nb–Ti and Nb–Zr can retain their superconductivity in fields ranging to well over 100 kG, and that, by suitable preparation, involving the introduction of high densities of extended structural defects or fine precipitates, very high supercurrent densities can also be achieved at these high fields. Several of the materials are available commercially, e.g. wires of $Nb_{0\cdot75}Zr_{0\cdot25}$, $Nb_{0\cdot67}Zr_{0\cdot33}$ and $Nb_{0\cdot6}Ti_{0\cdot4}$ in a heavily cold worked condition and the compound Nb_3Sn,

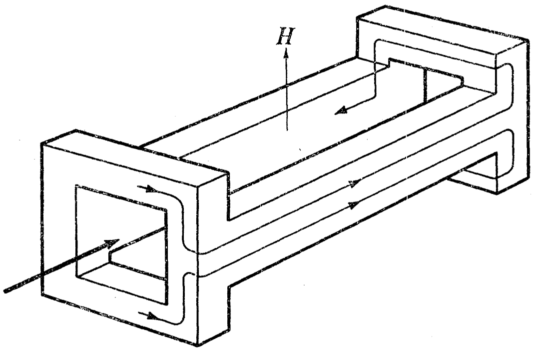

FIGURE 1. Schematic form of windings for an m.h.d. generator: —>—, current flow; ——>, path of combustion products, combustion chamber and duct omitted for clarity.

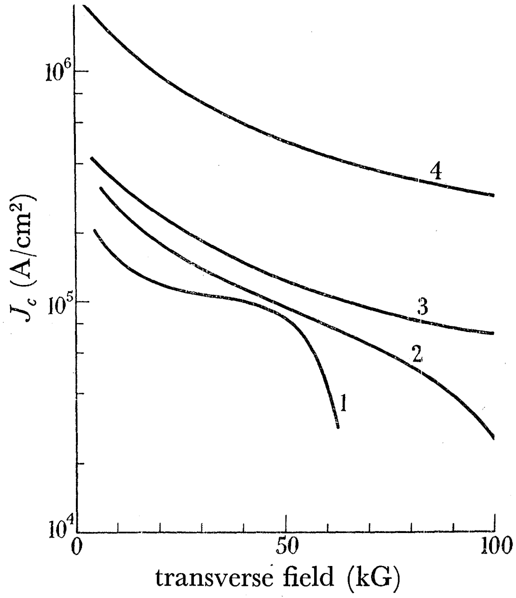

FIGURE 2. Current density as a function of transverse applied field for some commercial superconducting materials at 4·2 °K. (1) $Nb_{0\cdot75}Zr_{0\cdot25}$ 0·010 in. diameter wires. (2) $Nb_{0\cdot4}Ti_{0\cdot6}$ 0·010 in. diameter wires. (3) Nb_3Sn vapour deposited on stainless steel strip, overall current density. (4) As (3) but calculated for Nb_3Sn deposit alone.

either deposited on a stainless steel tape by a vapour-phase reaction or formed on the surface of niobium wire or tape by reaction with molten tin. The characteristics of short lengths of typical products are shown in figure 2. With them it becomes possible to contemplate the design of magnets operating at current densities of tens of kA/cm^2, at fields up to 100 kG.

ECONOMICS

Deferring, for a moment, consideration of technical feasibility, we may ask which of the types of winding outlined above is likely to prove most economic for an m.h.d. magnet. The answer involves consideration of both capital cost, including the cost of any necessary refrigeration, and the cost of power consumed. For resistive magnets, the balance between these quantities is a matter of choice: a high current density will result in a compact, low capital cost, magnet with high power consumption; a low current density in a large expensive magnet that is relatively cheap to run. In the case of a superconducting winding the capital cost will be related inversely to the current density achievable in the superconductor. It is convenient, therefore, to make an economic comparison between the systems in terms of current density as is done in figure 3, taken from unpublished work of Dr Norris and the author at C.E.R.L. and appropriate for the case of a 50 kG magnet for a generator with a thermal input of 1 GW. For simplicity, the annual cost is taken to be the actual annual running cost plus 10% of the initial capital investment. The width of the bands represents the uncertainty in estimating costs. In particular the upper limit of the superconducting band assumes the present-day small-quantity price for ductile niobium alloys; the lower limit is based on realistic, large-quantity, prices appropriate to the 1970's. Clearly, superconductors offer very considerable economies if they can be operated in coils at current densities greater than 2×10^4 A/cm^2—a value well within the

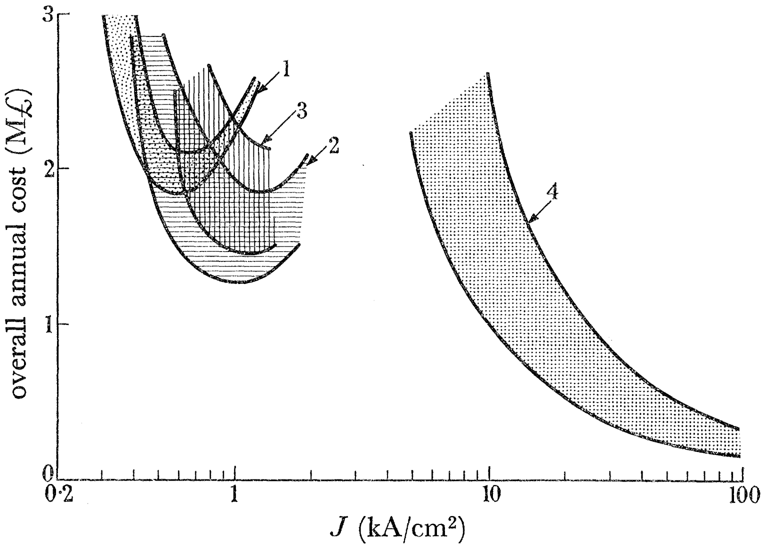

FIGURE 3. Overall annual cost of a magnet for a 1 GW input m.h.d. generator for four types of winding as a function of current density. (1) Copper at 55 °C; (2) high purity aluminium at 21 °K; (3) high purity sodium at 8·5 °K; (4) niobium-zirconium superconductor at 4·2 °K.

range of short sample performance illustrated in figure 2. Of greater significance is the conclusion that the high annual cost of any of the resistive windings would completely nullify the advantages of m.h.d. generation over conventional generation. The question of the feasibility of large superconducting magnets thus assumes considerable importance.

THE BEHAVIOUR OF SUPERCONDUCTORS IN COILS

Almost from the outset, the behaviour of the new, high field, superconductors when wound into coils proved disappointing. Wires, which would sustain a supercurrent density of, say, 10^5 A/cm^2 when tested in short lengths in modest externally applied fields, would carry no more than one-third or one-fifth of this when wound into a coil to generate the same field. Unwound again, they performed as originally. This type of behaviour, since called 'degradation', proved to be almost universal, worse with some materials than with others, more marked with split-coil systems than with simple solenoids and only absent at fields very close to the limit of the material. Although slight improvements were obtained by copper plating the wire, degradation appeared to become progressively worse with increasing coil size, and at one time gave rise to serious doubts as to the possibility of large coils at all (Berlincourt 1963).

A further peculiarity of simple superconducting coils, particularly those wound with unplated wire, is that their performance, although degraded in the manner described, is affected by the coils' previous magnetic history and can sometimes, but not always, be improved by repeatedly driving the coil into the normal state by exceeding its critical current. This 'training' procedure rarely brings the coil to the expected, short-sample, performance and has to be repeated each time the coil is cooled down (Le Blanc 1961).

It has also been observed that the flux in a coil does not always increase smoothly with current but frequently changes abruptly in small random steps—'flux-jumps' (Le Blanc 1963; Riemersma, Hulm & Chandrasekhar 1964). Moreover, superconducting coils were found to be sensitive to a variety of disturbances, vibrations, fluctuations in current and the like, any of which might trigger the return of the normal state, an event not without its hazards to the coil (Hulm, Chandrasekhar & Riemersma 1963; Martin, Benz, Bruch & Rosner 1963).

During this period the design of superconducting coils was very much a trial and error process and one could never be sure of the performance of a new type of coil until it had been wound and tested. In such circumstances superconductors could scarcely find engineering acceptance and many were stimulated to inquire into the nature of 'degradation', 'training' and 'flux jumping' (Kim, Hempstead & Strnad 1963*a*, *b*; Chandrasekhar, Gauster & Hulm 1963; Lubell, Mallick & Chandrasekhar 1964; Lubell & Mallick 1964; Le Blanc 1963; Gauster & Freeman 1964; Riemersma *et al.* 1964; Schrader & Kolondra 1964). It can now be seen that all three phenomena have a common origin in the nature of the current distribution in hard superconductors. In the next sections we shall examine the physics of these processes and shall find that there is scope both for tackling the problem at its root and for circumventing it completely.

The nature of high field superconductors

The essential feature of the superconducting materials that we shall be concerned with ('hard, type II' superconductors) is that they permit currents to flow, and large magnetic fields to penetrate, substantially into the bulk of the material, in sharp contrast to the more familiar ('soft, type I' superconductors) such as tin and lead, which permit fields and currents to penetrate only a surface layer some few hundred ångstroms thick. The substantial penetration of magnetic fields and the persistence of superconductivity in very large fields are now seen to be a consequence of the short 'coherence length' of the superconducting electrons in type II superconductors. The formal theory of such materials is now very extensive and has recently been reviewed (Goodman 1964; Lowell 1965). However, a 'soft' (i.e. homogeneous and defect-free) type II superconductor is not capable of supporting net bulk currents (Kamper 1963; Heaton & Rose-Innes 1964 *a*, *b*), and indeed, in order to obtain high bulk supercurrent densities in these materials, a high concentration of physical defects or inhomogeneities must be introduced (see, for example, Livingston & Schadler 1964). A plausible interpretation of these facts is that some restraining force is required to prevent the magnetic flux in the superconductor from moving under the influence of the Lorentz force exerted on it by the current (Gorter 1962; Goodman 1962; Anderson 1962) and it has been shown from thermodynamic considerations that inclusions and inhomogeneities (commonly called 'pinning centres') are capable of providing this restraint (Anderson 1962; Friedel, DeGennes & Matricon 1963; Silcox & Rollins 1963). The maximum supercurrent density J at any point is thus determined by a force equation which, in its most elementary form, is written

$$J_c \times B = \alpha_c, \tag{1}$$

where B is the local magnetic induction and α_c is a structure-sensitive 'pinning parameter' the effective value of which decreases with increasing temperature (Anderson 1962; Kim, Hempstead & Strnad 1962). If this equality is exceeded by increasing J or B, flux moves or creeps under the action of the excess force for as long as the imbalance remains (Kim *et al.* 1963 *b*). As will be seen below this is a dissipative process and the superconducting state is therefore limited by a surface of critical creep rate in a three dimensional plot of current density, magnetic induction and temperature of the general form shown in figure 4. As a magnetic field is applied to such a material, diamagnetic screening currents flow to oppose the entry of flux, starting at the surface of the body (Bean 1962; London 1963; Kim *et al.* 1963 *a*). As the critical condition, equation (1), is exceeded at each point, flux and current penetrate further into the body.

Stages in this process are depicted in figure 5. Here we have a semi-infinite plane block with a magnetic field applied parallel to its surface. Penetration of the field is opposed by induced currents which penetrate more deeply into the superconductor as the field is increased. At any stage, the depth of penetration, D, is given by Ampere's law

$$H_0 = 4\pi \int_0^D J_c(x)\, \mathrm{d}x. \tag{2}$$

Direct experimental confirmation of such penetration has recently been obtained on cylindrical specimens by Coffey (1965).

With a wire or a strip in an applied magnet field, oppositely directed currents penetrate from either surface, to shield the interior from the field and a large circulating diamagnetic current is set up (Bean 1962). Any net transport current carried by the wire is simply the difference between the forward and backward circulating current flow (London 1963).

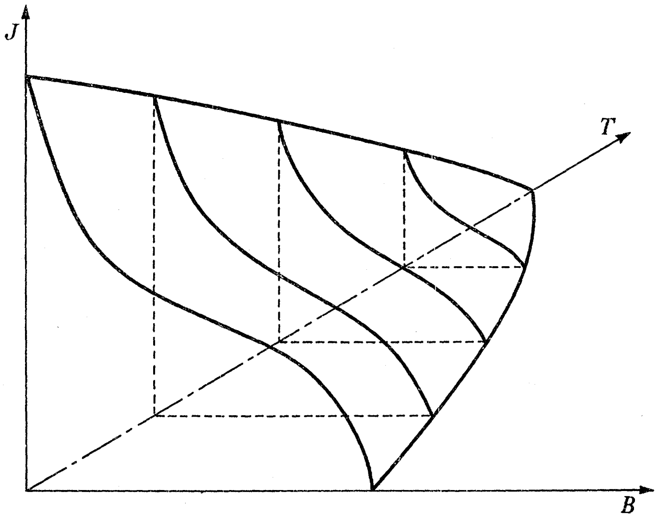

FIGURE 4. The critical surface defining the superconducting state of a hard superconductor in terms of current density J, magnetic induction B and temperature T.

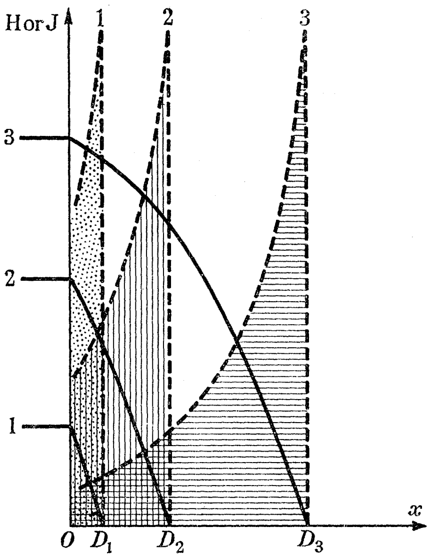

FIGURE 5. Stages in the penetration of flux and current into a semi-infinite hard type II superconductor exposed to an increasing magnetic field parallel to its surface. Conditions 1, 2 and 3 correspond to successively higher values of field at the surface. ---, current density; ——, magnetic induction.

During the early stages of energizing a superconducting solenoid, the net current flowing in the wire is considerably less than the circulating current. As the field and current are increased, however, the proportions change until, at the critical condition the forward current occupies the entire cross-section of the wire.

It follows from the concept of flux pinning that the process of current penetration is not reversible and that all changes in current must proceed from the surface. In consequence, the current distribution for decreasing current and field exhibits a hysteresis and a remanent circulating current remains when the field is reduced to zero.

The origins of degradation

It will be apparent from the figures and the above discussion that, in the process of increasing or decreasing the current in a hard type II superconductor, it is necessary for lines of flux to pass across lines of current. This is just the flux creeping process outlined earlier and it induces local e.m.f.s in a direction to oppose the change in flux linkage, i.e. to increase the local current density. Now by hypothesis the latter is already at its maximum value at each point and the electrical work done can only be dissipated.

With reference to figure 5, consider an element of current $J_x\,\mathrm{d}x$ at a depth x from the surface and let the field at the surface increase by an amount $\mathrm{d}H_0$. The change in flux linked with the element, per unit area of surface is

$$\int_x^D \mathrm{d}B_x\,\mathrm{d}x,$$

where B_x is the local value of the magnetic induction. The e.m.f. acting on the current element is then

$$\int_x^D \frac{\mathrm{d}B_x}{\mathrm{d}t}\,\mathrm{d}x$$

and the dissipation over the time interval $\mathrm{d}t$ is

$$J_x\,\mathrm{d}x\int_x^D \mathrm{d}B_x\,\mathrm{d}x.$$

Summing over all current elements we obtain for the dissipation per unit area of surface

$$\mathrm{d}W=\int_0^D J_x\,\mathrm{d}x\int_x^D \mathrm{d}B_x\,\mathrm{d}x. \tag{3}$$

For the simple case where J is independent of B, and therefore of x, we obtain

$$\mathrm{d}W=\tfrac{1}{2}JD^2\,\mathrm{d}H_0.$$

It will be noted that the dissipation *per unit volume* increases with increasing depth of penetration since both the current linked and the total flux involved increase with D.

Applying similar reasoning to the process of increasing the current in a superconducting coil we see that this necessarily generates heat in the superconductor and that the incremental dissipation, $\mathrm{d}W/\mathrm{d}H$, in a given conductor will be greater near the point of maximum total current flow. The *net* current at which this occurs depends both on the thickness of the conductor and its previous magnetic history. Moreover, the intensity of the dissipation increases with increasing layer thickness or wire diameter for a given J_c. These points are illustrated in figure 6 where is plotted the relative dissipation per cubic centimetre at equivalent points in the windings of three strip-wound solenoids, subject to the same rate of rise of centre field and magnetically identical except for strip thickness. It will be noted that the magnitude of the dissipation and the net current at which it reaches a maximum both decrease with conductor thickness.

The processes outlined above are inherently liable to thermal instabilities. The penetration of flux, due to an increment of field, liberates heat and raises the internal temperature of the superconductor. J_c is thereby decreased and the current has to penetrate more deeply into the material. This in turn entails further penetration of flux and further dissipation. Whether or not a new, stable, current and flux distribution is established depends on the interplay of such factors as the rate of rise of current and field, the specific heat and density of the material, the value of J_c and its dependence on field and temperature and the form of the initial current distribution, which in turn depends on the previous magnetic history. Distributions arising from a reversal of field, which can produce adjacent regions with oppositely directed flux, may be particularly troublesome because of the extra energy liberated by the mutual annihilation of flux quanta of opposite signs (Beasley, Fietz, Rollins, Silcox & Webb 1965). Against this background it can be inferred that the flux jumps observed in the course of energizing a solenoid are minor thermal instabilities which do not give rise to excessive temperature rise and are self-healing. It would appear that in most coils a major instability occurs at some current less than the critical current of the wire and such that the temperature rise is momentarily too great for the supercurrent to be supported using the *whole* available cross-section with the result that the material reverts to the normal state. As the current is being supplied by an external source, it continues to flow with consequent Joule heating and propagation of the normal region. In relation to the *bath* temperature such a transition has, of course, occurred prematurely, i.e. at a value of field and coil current lower than the critical values appropriate to a short sample at the bath temperature. Hence the name ‘degradation’.

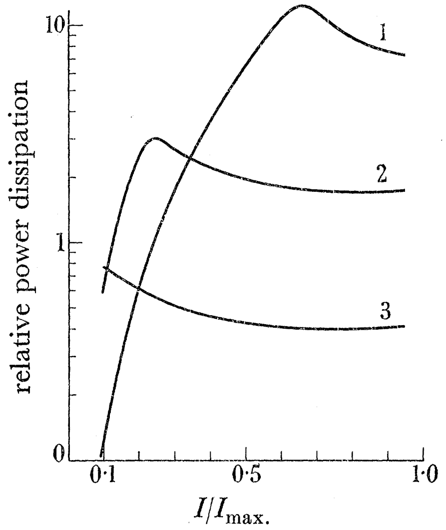

FIGURE 6. The relative dissipation per cubic centimetre at equivalent points in the windings of three strip-wound solenoids, subject to the same rate of rise of centre field and magnetically identical except for strip thickness t_1, t_2, t_3 are in the ratio 20:4:1. J_c is taken to be of the form $J_c = A(H=B)$ where A and B are constants.

There is much experimental evidence to support this picture. Measurements confirm that the magnetization per unit volume of hard superconductors does increase with increasing cross-section (Swartz 1962). The effect of previous magnetic history is shown up particularly well by the work of Lubell & Mallick (1964) in which the critical current of a small coil was reduced quite drastically by exposing it temporarily to an externally applied reverse magnetic field before energizing. The importance of specific heat is confirmed by experiments on porous sintered Nb_3Sn (Goldsmid & Corsan 1964; Hancox 1965), in which the stability of the material is much decreased when liquid helium is excluded from the pores. Plating wire with copper or silver makes some improvement which can be attributed to damping of rapid flux jumps as well as to the alternative current path which it provides (Riemersma *et al.* 1964; Schindler & Nyman 1964). The rate of rise of field is certainly important and, if too rapid, lowers the critical current (Laverick 1964).

These considerations, however, are not sufficient as they stand to explain degradation for it will be noted that all the factors involved are present in the case of tests on short samples. Indeed, non-catastrophic flux jumps can sometimes be observed in short sample tests (Schindler & Nyman 1964) and enough short sample tests have now been carried out to eliminate the possibility that 'weak spots' in the wire are responsible for poor coil performance (Berlincourt 1963). Evidently there is some feature characteristic of the coil environment and absent in short-sample tests, that precipitates or aggravates thermal instabilities. This factor has been the subject of much research.

It was pointed out at an early stage that the usual short sample test at constant field and increasing current does not reproduce the simultaneous increase of current and field experienced by a wire in a coil. A coil simulation test was accordingly devised in which a sample could be exposed to increasing current and field in ratios appropriate to different parts of a coil (Rosner & Schadler 1963). Although initial results were encouraging and the rate of increase of field is important, it must now be admitted that this does not provide the key to degradation. Poor thermal contact between the inner windings and the helium bath is evidently not crucial since short sample tests on wires embedded in grease or plastic show no degradation at slow rates of field sweep (Laverick 1964). Another suggestion was that the wire in a coil is bathed in the fluctuating field generated by flux jumps in other parts of the winding. The partial reduction in degradation brought about by copper plating the superconductor and interposing copper screens between layers (Schindler & Nyman 1964; Schrader & Kolondra 1964) lends support to this view as do the recent experiments of Cornish & Williams (1965). In the latter it was shown that a short sample of Nb–Zr wire would show degraded behaviour if it were located in a gap in the windings of the superconducting coil used for testing rather than in the open bore. Furthermore, this degradation was eliminated by inserting the sample in an electrically isolated copper tube. Unfortunately further work by the same group has failed to detect any field fluctuations in the appropriate frequency range of an amplitude sufficient to cause degraded behaviour.

A further ingenious possibility has been explored by the same group (Taquet 1965). They noted that long thin solenoids do not degrade and that short squat coils seem to be the most susceptible. Now, in a short non-superconducting solenoid the field in the windings takes the form shown in figure 7 with a circular region of zero total field on the mid-plane some distance from the outer radius (Boom & Livingston 1962). However, in a superconducting solenoid, because of the diamagnetism of the windings, the locus of zero field lies initially much closer to the outer radius. As the field is increased and the diamagnetism of the windings decreases, the locus of zero field must move radially inwards with the result that some points in the mid-plane experience a reversal of field. To see if this might give rise to degradation, Taquet simulated this situation with short samples by exposing them to large reverse fields before testing. He found that degradation could be induced but only with reverse fields an order of magnitude greater than could possibly exist in the relevant portion of a coil.

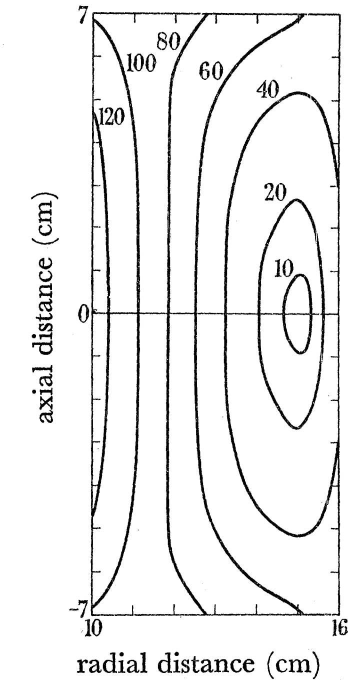

FIGURE 7. Contours of constant total field in one side of a solenoid. Outer diameter = 1·6 times inner diameter. Length = 0·7 times inner diameter. Field at solenoid centre 100, arbitrary units. (After Boom & Livingston 1962.)

Although the details of the process have yet to be demonstrated, there can be little doubt that degradation is related to the diamagnetism of the coil as a whole. For example, it is common experience that the proximity of another coil (as in a split pair) decreases the maximum coil current (Lucas, Stekly, de Winter & Laverick 1964). There is some evidence from work at the Culham Laboratory and elsewhere, that decreasing the over-all diamagnetic moment of a coil by using less superconductor and more insulation improves the coil performance of the wire (Lubell 1965). Moreover, maximum degradation is observed at low and intermediate fields where the diamagnetic moment is greatest. This effect is well illustrated in figure 8 (presented by J. C. Laurence at a conference on High Magnetic Fields at Oxford, July 1963, but unpublished) which shows a pronounced minimum in the critical current of flat coils of Nb_3Sn ribbon in small applied fields. Such minima can be avoided by immersing the coil in an externally applied field before it is energized, a process which reduces the magnetization (Schrader, Freedman & Fakan 1964). In practice this can be achieved by winding the coil in separate concentric sections and energizing them in sequence starting from the outermost (Schrader & Kolondra 1964; Coffey *et al.* 1965).

Precisely how the coil diamagnetism precipitates larger instabilities than occur in short-sample tests has not yet been established. One, as yet unexplored, possibility which seems compatible with all the evidence cited is as follows. Certain regions of the winding, away from the mid-plane, will experience large fields which *rotate* slightly about the axis of the wire as the current is increased, due to the changing diamagnetism of the coil and the movement of the zero field point. This will result in significant redistribution of flux and current in some of the wires and possibly in the juxtaposition of regions of oppositely directed flux. This particular perturbation is most unlikely to be experienced in the most widely used short-sample test, in which a hairpin-shaped sample is mounted along the axis of a solenoid, and may well prove to be a rather potent cause of thermal instabilities. This mechanism is consistent with the observation that the transition in a superconducting coil does not initiate at the point of maximum field (Riemersma *et al.* 1964; Donadieu 1965).

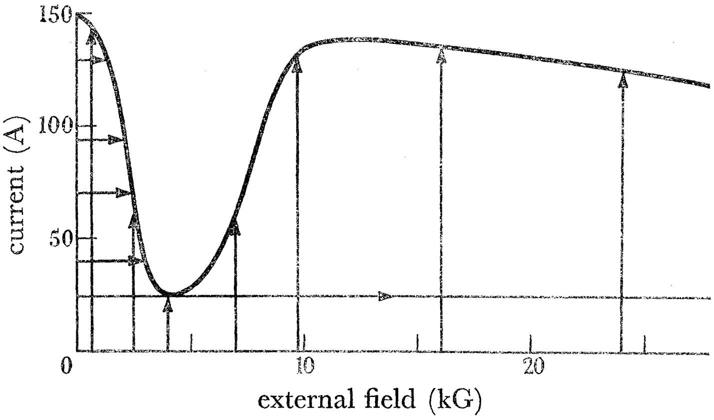

FIGURE 8. Critical current of a flat coil of Nb_3Sn tape as a function of external field. Degradation is apparent when the current is increased from zero at intermediate fields but can be avoided by applying a large magnetic field before passing current. (After Laurence 1963.)

Once it is accepted that the initial, circulating, current distribution in a superconductor affects the stability and dissipation during subsequent increases of current and field, the phenomenon of training becomes understandable. On a gross scale the winding as a whole acquires a diamagnetic moment as a result of the first increase of current to the (degraded) critical value and this modifies the detailed field and circulating current distribution through the wires during the next increase of current (Aron 1964). Moreover, the portion of the winding which goes normal at each transition later cools down in the remanent field produced by the rest of the windings and acquires yet another pattern of circulating currents (see Lubell 1966; Riemersma *et al.* 1964). These new distributions may be more or less favourable to subsequent dissipation—niobium–zirconium coils generally show a systematic improvement over the first few transitions; niobium–titanium coils, however, have been observed to give completely random values of critical current over a hundred or more transitions with no systematic trend of improvement.

The discussion so far has assumed a uniform distribution of pinning centres in the superconductor. This is not likely to be true in practice and it might be expected that nonuniform materials would be more liable to instability than uniform ones. Certainly, materials vary considerably in their susceptibility to flux jumping. Vapour-deposited Nb_3Sn can be manufactured in a form of high current density, which shows considerable instability even in short sample tests, or in a form of lower current density which is much more stable (Schrader & Kolondra 1964). Heat treatment of Nb–Zr alloys increases both their current density and their instability (Wernick 1962; Walker, Stickler & Werner 1963; Stekly & Zar 1965). The relation between structural non-uniformity, high critical current density and instability is still under study. Suffice it to note that instability is to be expected even in a uniform hard superconductor.

It would appear, on the basis of the above discussion, that the thermal instability responsible for degradation could be prevented or reduced by such means as using finer wires to reduce the dissipation, or artificially increasing the thermal capacity of the wire (with suitable additional material or by raising the temperature) to reduce the temperature rise or, if the rotation of field patterns is indeed important, by the use of suitable ferromagnetic insets or compensating windings. Work is indeed proceeding along these lines and, no doubt, satisfactory solutions will be found. In the meantime, however, an engineering approach has won the day—at least temporarily.

STABLE COILS

The engineering approach to the problem of degradation is to accept the presence of thermal instabilities in superconductors and to ask what can be done to prevent the resulting transient normal regions from propagating. The first steps in this direction were taken last year by Laverick (1965) and Laverick & Lobell (1965) who cabled together superconducting wires and copper wires, using indium filling to secure good electrical contact, and wound large coils from such cables with very good access for liquid helium. These coils, which had stored energies up to 600 kJ, showed noticeably less degradation and were considerably more stable than any previous coils of similar size. The improvement resulted from the presence of the continuous alternative low resistance current path which, when necessary, could relieve transient normal regions of part of their current, the good heat transfer to the helium minimizing the temperature rise of the copper and indium.

The Avco Laboratories, working on the same lines, made a thorough analysis of this technique and have pursued it to a very successful practical outcome (Stekly & Zar 1965; Kantrowitz & Stekly 1965). They argue that the basic criterion for completely stable operation at any point is that the temperature rise, when all the current is flowing in the alternative low resistance path, or 'substrate', must not exceed the critical temperature, T_c, of the superconductor appropriate to the current and magnetic field in question.

Thus

$$J_B^2 \rho A < h(T_c - T_b)P, \tag{4a}$$

or

$$J_B < \{h(T_c - T_b)k/\rho\}^{\frac{2}{3}}\, I^{-\frac{1}{3}} = SI^{-\frac{1}{3}}, \tag{4b}$$

where J_B is the current density in the substrate, ρ is the resistivity of the substrate, A is the cross-sectional area of the substrate, h is the heat transfer coefficient, T_b is the temperature of the bath, and P is the cooled perimeter $= k\sqrt{A}$. In order to design a magnet winding from relation (4) above, it is necessary to know h for the particular arrangement proposed and T_c (and ρ) for the field and current in question.

If the inequality can be satisfied, current will always return completely to the superconductor after an instability, i.e. transient normal regions will collapse at all points along their length. To demonstrate this, Kantrowitz & Stekly (1965) wound a magnet with a composite conductor, consisting of a copper strip of low resistivity in which were embedded nine wires of heat-treated Nb–Zr alloy, allowing free access for liquid helium to all the turns. Some characteristics of this magnet, which showed no degradation, are given in the paper by Brogan in this volume.

A similar approach being pursued independently at the Culham Laboratory (Cornish 1966; Williams 1965) stems from earlier work on the velocity of propagation of a normal zone in a superconducting wire (Broom & Rhoderick 1960; Cherry & Gittleman 1960). This analysis indicates that such a region will collapse from its ends (but not spontaneously at all points along its length as in the Avco criterion) if

$$J^2\rho A < 2h(T_c - T_B)\,P, \tag{5}$$

and will propagate from its ends if the reverse holds true. A simple method of determining whether a given composite structure satisfies inequality (5) is to produce with a heater a normal region in a short sample of it when under the required conditions of field, current and thermal environment and see whether or not the normal region propagates. Such tests have been used successfully in the design of stable non-degrading coils based on stranded composite cable (Cornish 1966).

There is no doubt that the use of composite conductors provides a complete solution to the problem of degradation and will allow the maximum economic use of the superconductor. In its simplest form, using relation (4) with freely boiling helium and copper of moderate purity, it is not a compact solution—a typical current density in the composite would be 6 kA/cm^2 falling to perhaps 3 kA/cm^2 over-all when allowance is made for cooling channels. Greater compactness will undoubtedly be achieved by exploiting relation (5) with high purity aluminium and possibly with forced cooling. In any case, compactness is not usually a problem with very large magnets, particularly those required for m.h.d. generators.

PROTECTION

Two risks attendant upon driving a simple superconductor coil into the normal state are damage to the superconductor and breakdown of the electrical insulation (Hulm *et al.* 1963). Both risks increase with the magnitude of the stored energy. The former arises from the high resistivity of hard superconductors when in the normal state ($\sim 30\,\mu\Omega$ cm) coupled with the very large current densities at which the wires operate. When a normal region develops in the winding, current continues to flow in it because of the inductance. During the decay of the current the energy dissipated in such a region may be sufficient to melt or even vaporize it. Such indeed was the fate of the first superconducting magnet to exceed 100 kG (Martin *et al.* 1963). If the current is reduced too rapidly, either because of rapid propagation of the normal region or in an attempt to minimize dissipation, the induced voltages can break down the electrical insulation of the coil.

The capital cost of a superconducting magnet for a large m.h.d. generator is likely to be of the order of £10 million and it is obviously necessary to design it so as to avoid damage by any deliberate or accidental return to the normal state. A number of methods of protection have been considered (Stekly 1963*a*, *b*; Smith 1963; Dowley 1964) but not all of them can be applied to stable coils. It is easy to visualize a failure of the refrigerator or of the thermal insulation, causing loss of liquid helium and necessitating a return to the normal state. A more dangerous situation would be the local blockage of a cooling channel resulting in the appearance of a *small* warm spot in one of the windings at full current. If this happens it must of course be detected and the current reduced as rapidly as possible. Even so it is vital to ensure that the dissipation in the warm spot during current decay is so limited as to prevent an unsafe temperature rise. The energy balance in any region that has lost all cooling is simply

$$J^2\rho\,\mathrm{d}t = \gamma C\,\mathrm{d}\theta, \tag{6}$$

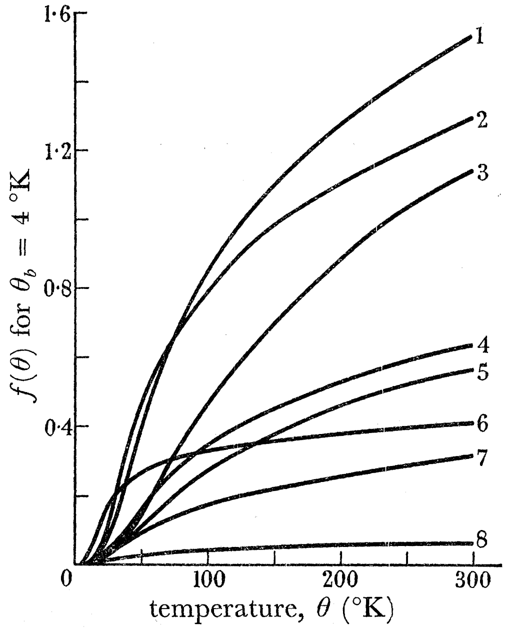

FIGURE 9. The function $f(\theta)$ (10^{-9} J cm^{-3} (Ω cm)$^{-1}$) (see equation (7)) plotted against maximum allowed temperature for a number of metals. Purity of the metal is indicated by its residual resistance ratio ρ (273 °K) : ρ (4 °K) in a field of 60 kG. (1) Cu (100:1); (2) Ag (100:1); (3) Cu (20:1); (4) Al (500:1); (5) Al (100:1); (6) Na (3000:1); (7) Zn (100:1); (8) In (500:1).

where $J(t)$ is the current density, $\rho(\theta)$ is the resistivity of the conductor, γ is the density of the conductor, $C(\theta)$ is the specific heat of the conductor per unit mass, t is the time, θ is the temperature, θ_m is the maximum temperature, and θ_b is the temperature of cooling bath. Hence

$$\int_0^\infty J^2(t)\,\mathrm{d}t = \int_{\theta_b}^{\theta_m} (\gamma C/\rho)\,\mathrm{d}\theta = f(\theta). \tag{7}$$

The two most effective ways of limiting θ_m, for a given value of J are: (i) to lower the effective resistivity of the superconductor (in the normal state) by embedding it in a matrix of low resistivity metal such as copper, aluminium or sodium; and (ii) to cause the current to decay as rapidly as is consistent with the dielectric strength of the electrical insulation. The effectiveness of different matrix materials in limiting θ_m is shown in figure 9 (Maddock, James & Chester 1965) from which can be read off the maximum permissible value of $\int_0^\infty J^2\,dt$ ($= \frac{1}{2}J_0^2\tau$ for an exponential current decay) for a given material and maximum desired temperature.

Now the value of τ for a large coil having only a small normal region in its windings would be disastrously long and it proves necessary in addition to shorten the time constant by auxiliary means. Our own studies indicate that the best method of doing this is for a large m.h.d. magnet is that illustrated in figure 10. When necessary, the power supply is isolated by the switch and the time constant is reduced, as far as is safe, by an appropriate choice of shunt-resistor—a typical value might be 3 Ω giving a time constant of 3 min or so. In this way a large fraction of the stored energy is dissipated outside the cryostat. It appears that it will be quite feasible to detect dangerous warm spots in the windings sufficiently rapidly and to develop the necessary high current switch capable of interrupting 10 kA at 20 kV.

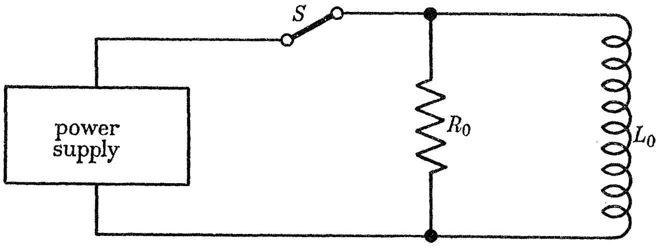

FIGURE 10. Protection circuit using a switch to isolate the power supply and a shunt resistor to control the current decay. A large fraction of the stored energy is dissipated in the resistor.

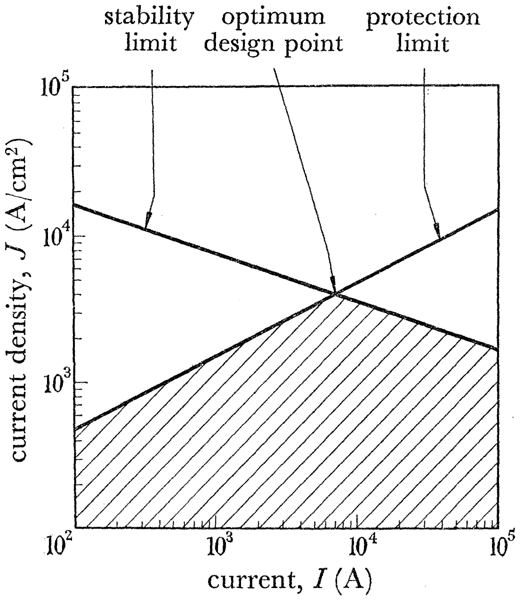

FIGURE 11. Limiting conditions for stability and protection plotted as relations between J and I for a large m.h.d. magnet (see equations (4) and (8)). The region of stable and safe operation is shown shaded.

The value of the series resistor, R, is chosen to satisfy the relation

$$IR = V_m$$

at the instant of switching, where I is the current in the conductor and V_m is the maximum voltage that the insulation can withstand. It can then be shown that the maximum safe operating current density in the windings is given by

$$J = \{f(\theta_m)\, V_m I/E\}^{\frac{1}{2}}, \tag{8}$$

where E is the stored energy in the magnet and it is assumed that $E = \frac{1}{2}LI^2$. The conditions for stability and protection (4) and (8) above, are illustrated in figure 11, for the case of a large m.h.d. magnet with a stored energy of $1{\cdot}5 \times 10^{10}$ J, in which $V_m = 20$ kV, with good copper (resistance ratio 100:1) used in the composite, and in which the maximum possible temperature rise is limited to room temperature.

Maximum current density in the substrate that is compatible with both requirements is obtained by choosing the operating current corresponding to the intersection of the pair of lines.

This optimum current density is given by

$$\begin{aligned} J_{\text{opt.}} &= \{V_m\, f(\theta_m)/E\}^{\frac{1}{5}}\,\{h(\theta_c-\theta_b)k/\rho\}^{\frac{2}{5}} \\ &= 3800\ \text{A/cm}^2 \text{ in this case} \end{aligned} \tag{9}$$

and the optimum current by

$$\begin{aligned} I_{\text{opt.}} &= \{E/f(\theta_m)\, V_m\}^{\frac{3}{5}}\,\{h(\theta_c-\theta_b)k/\rho\}^{\frac{4}{5}} \\ &= 7200\ \text{A in this case.} \end{aligned} \tag{10}$$

Since the current density in the superconducting component is likely to be in the region of 10^5 A/cm^2 it can be seen that the requirements of protection, in addition to those of stability, dictate a very large ratio of low resistance metal to superconductor in the composite.

It can be seen that a composite conductor is necessary for stability and protection in a large magnet and that it can be specified in some detail even at this stage. Indeed the Avco Corporation and the C.E.G.B. are both proceeding with the construction of intermediate sized composite magnets with stored energies in the region of 10^7 J.

Future developments

The considerations of stability and protection discussed above have important consequences for the development of improved superconducting materials. When a very compact winding is required, and full use is made of high purity metals and forced cooling, superconductors of very high current density are worth pursuing no matter how unstable they may prove to be. For the very large magnet, where compactness of winding is not important, and over-all cost is, the current density taken over in the matrix is likely to be relatively low and the chief requirement of the superconductor is that it should provide ampere turns at the lowest cost, again no matter how unstable it proves to be by itself. An important factor in the latter case will be the ease with which superconductors can be combined with high purity metals in a cheap production process which allows the properties of both components to be optimized. Here the ductile niobium alloys have an obvious advantage over the brittle compounds, although recent progress in flame-spraying Nb_3Sn on to a copper substrate may well bring this more powerful material into the engineering arena.

On the mechanical side there is still much detailed work to be done before a full sized m.h.d. superconducting magnet can be designed. Data will be required on heat transfer in long channels of liquid helium, and on the dielectric strength of insulating materials at low temperatures. The magnetic forces acting on the windings will be very large—up to 250 tons/ft.^{2} outwards on the side faces and, in places, up to 1000 tons/ft.^{2} pinching the windings together. These values can certainly be sustained with existing materials at low temperatures but they open up a whole range of interesting engineering problems such as whether to have the support structure all at 4 °K or, alternatively, how best to support the loads with minimum heat leak into the liquid helium. The refrigeration requirements will depend on the design of support structure but they might well be determined by the time one is willing to spend waiting for the magnet to cool down.

Conclusions

Superconducting magnets will be an economic necessity for future central m.h.d. generators. The hitherto serious problems of degradation, training and instability in superconducting coils can now be circumvented and the problem of protection solved by techniques which have as a common factor the combination of superconductor with large proportions of low resistance metal. A number of interesting engineering problems are emerging but none of them appears to be outside the scope of existing technology. The design of large superconducting m.h.d. magnets can now proceed with assurance.

The achievement of very compact windings for smaller applications will entail the prevention of thermal instabilities in the superconductor and means for ensuring this are now becoming clear.

The author is grateful to his colleagues both within the C.E.G.B. and outside it for communicating unpublished results.

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XIX. Discussion

W. T. Norris (Central Electricity Research Laboratory)

It is instructive to calculate the leakage currents that are to be expected flowing directly from one electrode to another when current flows uniformly from infinity in the upper half plane to a row of equilength equispaced plane electrodes on the real axis, and when there is a Hall effect. This is the situation discussed in the first part of Dr Haines's paper (p. 440 above) which also quotes the transform given by Witalis from the Z plane under consideration to a w plane in which effective equipotentials and current flow lines are straight and conditions are undisturbed, or at most uniformly slid, infinitely far from the electrodes.

The transform may be written

$$\frac{\mathrm{d}w}{\mathrm{d}Z} = \mathrm{e}^{-\mathrm{i}(\theta-\psi)}\left[\frac{\sin\{\frac{1}{2}\pi(Z/\rho-b)\}}{\sin(\frac{1}{2}\pi Z/\rho)}\right]^{-\phi}\frac{\sin\frac{1}{2}\pi(Z/\rho-a)}{\sin(\frac{1}{2}\pi Z/\rho)}, \tag{1}$$

where θ is the Hall angle, ψ is the angle between the direction of current flow at infinity and the perpendicular to the electrode line, $b\rho$ is the insulator width and 2ρ the electrode width and where

$$\tfrac{1}{2}\pi a = \theta-\psi+\tfrac{1}{2}\pi b\,\phi \quad\text{and}\quad \phi = \tfrac{1}{2}-\theta/\pi.$$

Figure 2 of Haines's paper illustrates both the original and transformed planes. This transform has already been used extensively by Witalis to calculate the influence of electrode segmentation on m.h.d. generator performance. If there is a leakage current then $a > b$. The magnitude of the leakage current per unit width of electrode is

$$I_L = J\cos\theta\ (\text{length of the fin in the } w \text{ plane}),$$

whereas the net electrode current is clearly

$$I_E = 2\rho J\cos\psi, \tag{2}$$

where J is the current density in the duct. The length of the fin is given by the integral along the real Z axis

$$\int_{Z=b\rho}^{Z=a\rho}|\mathrm{d}w| = \int_{Z=b\rho}^{a\rho}\left|\frac{\mathrm{d}w}{\mathrm{d}Z}\right|\mathrm{d}Z.$$

Putting

$$s = \frac{\sin\{\frac{1}{2}\pi(Z/\rho-b)\}}{\sin(\frac{1}{2}\pi Z/\rho)}$$

we can write the fin length as

$$\frac{2\rho}{\pi}\int_0^q\frac{\mathrm{d}s}{s^\phi}\sin(\tfrac{1}{2}\pi a)\,\frac{q-s}{1-2s\cos(\frac{1}{2}\pi b)+s^2}$$

with

$$q = \frac{\sin\frac{1}{2}\pi(a-b)}{\sin\frac{1}{2}\pi a},$$

which can always be less than unity.

The part consisting of the reciprocal of a quadratic can be expanded as a sine series and the whole expression may be integrated so that the leakage current can be rewritten as

$$J_L = \frac{2\rho J\cos\theta}{\pi}\,\frac{\sin\frac{1}{2}\pi a}{\sin\frac{1}{2}\pi b}\,q^{1-\phi}\sum_{n=1}^{\infty}\frac{\sin(\frac{1}{2}n\pi b)\,q^n}{(n-\phi)\,(n-\phi+1)}. \tag{3}$$

This series converges as $1/n^2$ beyond the $(1/b)$th term and is convenient for machine computation. However, each term of the sum may be split up thus

$$\frac{q^n}{(n-\phi)\,(n-\phi+1)} = \frac{1}{n^2}+\frac{(2\phi-1)n+\phi(\phi-1)}{n^2(n-\phi)\,(n-\phi+1)}-\frac{(1-q^n)}{(n-\phi)\,(n-\phi+1)},$$

and the first part summed separately: further subdivision is possible but for the case of most interest, i.e. small values of b, is pointless. Then the leakage current can be written

$$I_L = J\cos\theta\,\frac{2\rho}{\pi}\,\frac{\sin\frac{1}{2}\pi a}{\sin\frac{1}{2}\pi b}\left\{\tfrac{1}{2}\pi b[1-\ln(\tfrac{1}{2}\pi b)]-\sum_1^{\infty}\frac{B_n(\frac{1}{2}\pi b)^{2n+1}}{2n\{(2n+1)\}}\right.$$
$$\left.+\sum_1^{\infty}\left[\frac{(2\phi-1)n+\phi(\phi-1)}{n^2(n-\phi)\,(n-\phi+1)}-\frac{1-q^n}{(n-\phi)\,(n-\phi+1)}\right]\sin\tfrac{1}{2}n\pi b\right\},$$

which for small values of b when only the logarithmic term is important becomes approximately

$$I_L = (2\rho/\pi)\,J\cos\theta\,\sin(\theta-\psi)\,\ln(2/\pi b). \tag{4}$$

For given angle of flow of current at infinity as the Hall parameter and angle increase from zero the leakage current goes through a maximum near $\theta=\frac{1}{4}\pi$. This is apparent from inspection of expression (4) and follows from expression (3) also. The leakage currents are not large and some typical values are shown in figure 1 for $\psi=0$, i.e. with current at infinity flowing normally to the electrode row. As ψ increases and the current in the main duct has an axial component opposite to that of the leakage current the

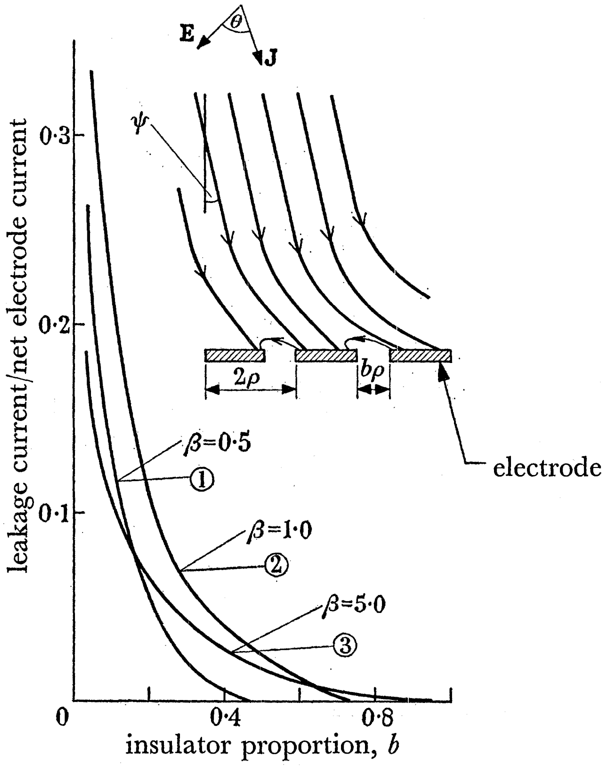

FIGURE 1. Dependence of ratio of leakage current to net electrode current on insulator width ($b\rho$) for various Hall parameters: (1) $\beta = 0{\cdot}5$, $\theta = 26{\cdot}56°$; (2) $\beta = 1{\cdot}0$, $\theta = 45°$; (3) $\beta = 5{\cdot}0$, $\theta = 78{\cdot}69°$; electrode width is $(2-b)\rho$; current at infinity flows perpendicular to the electrode wall ($\psi = 0$). In the sketch the magnetic field is out of the paper.

leakage current decreases: as ψ decreases the leakage current increases. Even for quite narrow insulators the leakage current is only a fraction of the net flow to an electrode.

In a practical generator this analysis will possibly be inadequate and then precisely in the region of greatest interest, that is close to the electrode surfaces where the gases will usually be colder and more resistive than in the main flow. This will probably lead to lower leakage currents than those calculated here. In all these events the effect on the electrical performance of generator will be slight, at least as far as fossil fuel fired generators are concerned where the Hall angle is not close to a right angle.

Certainly very heavy current between neighbouring electrodes may aggravate itself in a catastrophic way and arcs between adjacent electrodes would harm both the electrodes and the insulator between. The formation of arcs must be avoided but the criterion for their absence seems to be more dependent on other factors, notably interelectrode voltage, than simply on these leakage currents.

Z. Croitoru (Électricité de France)

In a m.h.d. generator, there should be a current component J_y perpendicular to the gas velocity and to the magnetic field. If the Hall effect is also present the presence of an electric field component E_x parallel to the gas velocity is implied. The continuity of these two components must be ensured on the duct surface and this has led to the proposal for segmented generators (figure 2).

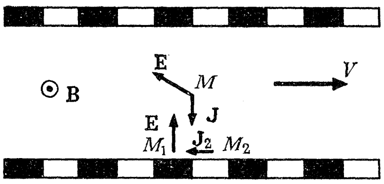

FIGURE 2

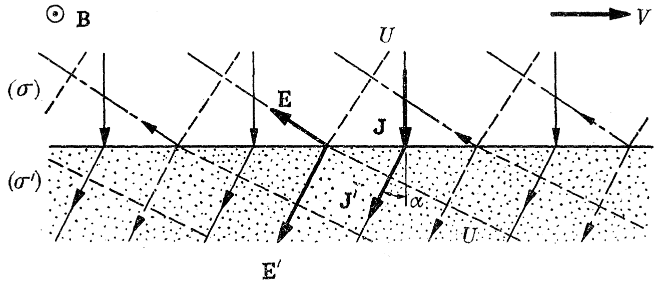

FIGURE 3

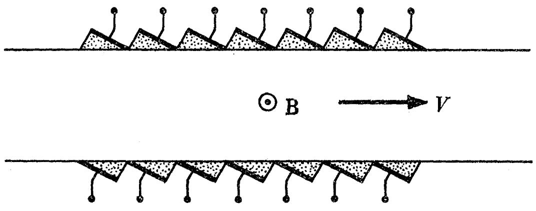

FIGURE 4

Actually, near the insulating segments, there can be no J_y component; therefore, near the conducting segments, the mean value of J_y will be higher than in the duct centre. Similarly, there can be no E_x component near the conducting segments and this will give a higher mean value of this component near the insulating segments. Moreover, as Dr Haines has shown, important non-uniformities are to be expected at high values of the Hall parameter β. The local increase in Joule losses is always harmful, but especially if non-equilibrium conductivity is to be expected.

The poor performance of segmented electrode generators is a result of the discontinuities at three very different media; (*a*) the gas, with finite conductivity σ and Hall effect; (*b*) the electrode segments, with practically infinite conductivity; (*c*) the insulating segments with practically zero conductivity.

A perfect solution should avoid such discontinuities. Current collecting walls should have neither zero nor infinite conductivity. Semiconducting electrodes with finite conductivity σ' of the same magnitude as the gas conductivity σ (Croitoru 1965) should be used. With such a material, the conditions of continuity of E_x and J_y may be respected at any point of the duct and the contact surface may be continuous (figure 3). The conditions of continuity show that current lines in the semiconductor make an angle α with Oy such that

$$\tan \alpha = \beta\sigma'/\sigma.$$

The equipotential planes in the semiconductor make the same angle with the Ox axis. Conducting electrodes can be placed in these planes without disturbing the current lines. Such electrodes are required for practical reasons, since a finite number of external loads are desirable.

Finally, the shape of m.h.d. generators using semiconducting electrodes should be that shown in figure 4. In such generators there are uniform conditions and no additional losses anywhere in the duct. On the other hand, there are additional Joule losses in the semiconducting material. Such losses are unavoidable in a generator demonstrating the Hall effect and having a finite number of load circuits. At least these losses are lower than in a segmented generator and have no effect on the internal behaviour of the generator.

Obviously, the different configurations of m.h.d. generators already considered for segmented electrodes, i.e. Faraday mode, series connected, Hall generators may also be operated with semiconducting electrodes.

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